Annuities and Loan Repayments

General Maths HSC

NAME: _____

HSC CAPACITY MATRIX – GENERAL MATHEMATICS

TOPIC: Financial Mathematics 5 – Annuities & loan repayments 3 weeks

CONTENT	CAPACITY BREAKDOWN!	DONE IT!!!!	GOT IT!!!!!	ON MY WAY!	WORKING ON IT!	HELP!!!!
1. Deep understanding of the term "annuity" 2. Calculation of the future value (A) of an annuity (or the contribution per period), using $A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$	Compound interest W/S Skillsheet 8.1 Ex 8A all even questions S/S task – Annuity calc					
3. Calculation of the present value (N) of an annuity (or the contribution per period), using $N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$ or $N = \frac{A}{(1+r)^n}$	Ex 8B all even questions					
4. Using tables to solve problems involving annuities	S/S task – Future and present value tables Ex 8C all even questions					
5. Use the present value formula for annuities to calculate loan instalments, and hence the total amount paid over the term of a loan	Ex 8D all even questions					
6. Investigate various processes for repayment of loans 7. Calculate the fees and charges for different options for borrowing money in order to make a purchase.	Types of loans task					

Your say!

What was the most important thing you learned?_____

What was something new you learnt? _____

What part(s) of this topic will you need to work on? _____

QUICK REVISION - COMPOUND INTEREST

Compound Interest formula – is it on the Formulae sheet?

SETTING OUT

eg Calculate the value of each of the following investments.

(a) $5000 invested at 7% p.a. for 4 years with interest compounded annually.

(b) $17 500 invested at 9% p.a. for 5 years with interest compounded six-monthly.

(c) $42 000 invested at 6.8% p.a. for 4 years with interest compounded quarterly.

INVESTIGATING ANNUITIES

But firstly, what is an ˝annuity˝?

Let´s investigate:

eg Laura invests $5000 at the end of each year at 8% p.a. for 4 years with interest compounded annually.

(a) Sketch a timeline.

(b) Find the amount to which the first $5000 will grow.

(c) If Laura invests another $5000 at the end of the second year, to what amount will this $5000 grow?

(d) A third $5000 is invested after another year. To what amount will this $5000 grow in one year?

(e) How much will Laura make on her investment?

(f) How much altogether?

A QUICKER METHOD PERHAPS?

- The value of an annuity is calculated by adding the value of each amount contributed as a separate compound interest investment. The formula below will calculate the value of an annuity:

This formula is given to you!

FUTURE VALUE OF ANNUITY FORMULA $\Rightarrow$ $$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

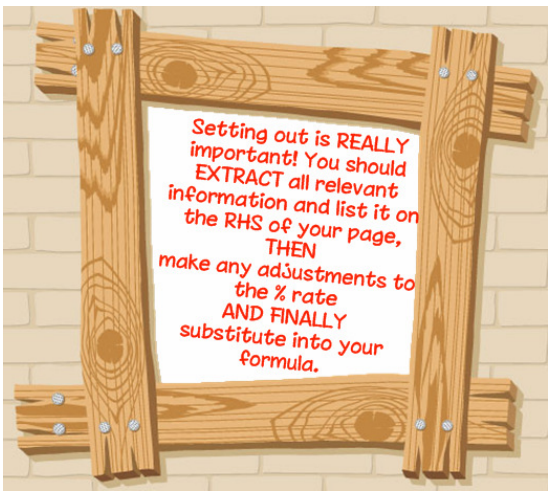
where M is the amount of each periodical contribution;
r is the interest rate per period expressed as a decimal;
n is the number of deposits.

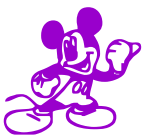
- The amount of each contribution to annuity to reach a certain goal can be calculated using the formula:

$$M = \frac{Ar}{(1+r)^n - 1}$$

This formula is NOT given to you – can you see how to change the above formula into this one?

eg Lance invests $1000 into a fund at the end of each year at 8%pa interest, for 10 years. Calculate the future value of the annuity after 10 years.





eg Belinda invests $500 in a fund every six month at 9.5% pa interest, compounding six-monthly for 15 years. Calculate the future value of the annuity after 15 years.

eg Luke wants to save $12 000 in the next five years. The best interest rate that he can obtain is 7.5% pa, with interest compounded annually. Calculate the amount of each annual contribution that Luke must make.

PRESENT VALUE OF AN ANNUITY

DEFINITION: The present value of an annuity is the single sum that can be invested under the same terms as annuity and will produce the same financial outcome.

The present value of an annuity can be calculated using the formula:

To compare an annuity with a single sum investment, you need the **present value of the annuity!**

$$N = \frac{A}{(1+r)^n}$$

when you know the future value of an annuity.

If you know the amount of each contribution of the annuity, you can calculate the present value using the formula

$$N = M\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right],$$

where **M** is the contribution period;

r is the interest rate per period (as a decimal);

and **n** is the number of interest periods.

Investments can be compared using the present value formula. The investment with the greater present value will produce the greater financial outcome over time.

eg Scott has an annuity that has a future value of $500 000 on his retirement in 32 years. The annuity is invested at 6.5%pa with interest compounded annually. Calculate the present value of Scott's annuity.

eg Tom has an annuity to which he contributes $1100 annually at 7.5%pa interest, compounded annually. The annuity will mature in 25 years. Calculate the present value of the annuity.

eg Which of the following investments would give the greater financial return?
INVESTMENT A: An annuity of $100 deposited per month for 20 years at 12% pa interest, compounding six monthly;
INVESTMENT B: A single deposit of $10 000 invested for 20 years at 12% pa with interest compounding six monthly.

USING TABLES...

A table such as the one below can be used to find the value of an annuity by multiplying the amount of the annuity by the future value of $1.

Future value of $1												
	Interest rate per period											
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524

eg Use the table to calculate the value of an annuity into which $8 000 is deposited at the end of each year at 7% pa interest, compounded annually for 11 years.

Present value of $1												
	Interest rate per period											
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	6.2065	5.9377

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.4924	6.1944
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.7499	6.4235
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.9819	6.6282
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	7.1909	6.8109
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	7.3792	6.9740
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.5488	7.1196
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.7016	7.2497
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.8393	7.3658
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.9633	7.4694

eg Phoebe invests $850 per year in an annuity at 6% pa for 9 years, with interest compounded annually. Use the table to calculate the present value of Phoebe's annuity.

LOAN REPAYMENTS

When a loan is taken out and is repaid in equal monthly instalments, the pattern of repayments works similar to an annuity. Each month interest compounds on the balance owing on the loan and then a repayment is made.

Consider the present value (N) formula:

$$N = M\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right]$$

To calculate the amount of each monthly repayment, we need to make M the subject of this formula.

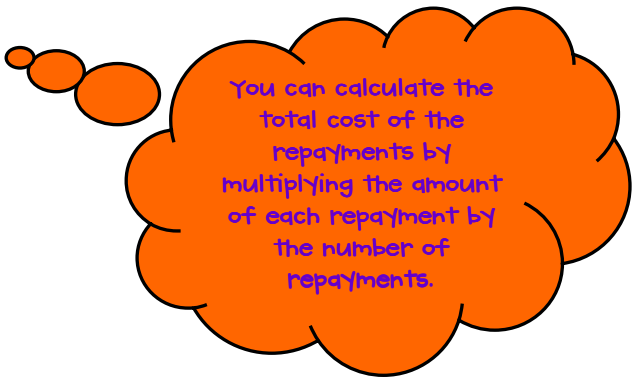
In this formula, M is the amount of each repayment, N is the amount borrowed, r is the interest rate per repayment period as a decimal and n is the number of repayments to be made.

Hmmmm....

- By considering the amount borrowed in a loan as the PRESENT VALUE OF AN ANNUITY, you can apply the present value formula to calculate the amount of each repayment.
- The total cost of the loan can be calculated by multiplying the amount of each repayment by the number of repayments to be made;
- The length of time that it will take to repay a loan can be calculated by using GUESS AND REFINE!

eg Calculate the monthly repayments on a loan of $45 000 to be repaid in monthly instalments over 14 years at an interest rate of 12.5% pa.

eg Calculate the total cost of repaying a $200 000 home loan at 9% pa in equal monthly repayments over a 25 year term.



eg A $100 000 home loan is taken out over a 25 year term at an interest rate of 12% pa reducible interest. The minimum monthly repayment on the loan is $1053.22. How long will take the loan to be repaid at $1200 per month?

2011 HSC MC

16 A loan of \$25 000 is used to purchase a car. The term of the loan is three years and the interest rate is 9% per annum, compounded fortnightly.

Which equation should be used to calculate the fortnightly repayments, M?

(A) $25\,000 = M\left\{\dfrac{(1+0.09)^3-1}{0.09(1+0.09)^3}\right\}$

(B) $25\,000 = M\left\{\dfrac{(1+0.09)^{78}-1}{0.09(1+0.09)^{78}}\right\}$

(C) $25\,000 = M\left\{\dfrac{\left(1+\dfrac{0.09}{26}\right)^3-1}{\dfrac{0.09}{26}\left(1+\dfrac{0.09}{26}\right)^3}\right\}$

(D) $25\,000 = M\left\{\dfrac{\left(1+\dfrac{0.09}{26}\right)^{78}-1}{\dfrac{0.09}{26}\left(1+\dfrac{0.09}{26}\right)^{78}}\right\}$

Q23

(c) An amount of \$5000 is invested at 10% per annum, compounded six-monthly. **2**

Compounded values of \$1

Period	Interest rate per period				
	1%	*5%*	*10%*	*15%*	*20%*
1	1.010	1.050	1.100	1.150	1.200
2	1.020	1.103	1.210	1.323	1.440
3	1.030	1.158	1.331	1.521	1.728
4	1.041	1.216	1.464	1.750	2.074
5	1.051	1.276	1.611	2.011	2.488
6	1.062	1.340	1.772	2.313	2.986

Use the table to find the value of this investment at the end of three years.

Q27

(d) Josephine invests $50 000 for 15 years, at an interest rate of 6% per annum, compounded annually. **4**

Emma invests $500 at the end of each month for 15 years, at an interest rate of 6% per annum, compounded monthly.

Financial gain is defined as the difference between the final value of an investment and the total contributions.

Who will have the better financial gain after 15 years? Justify your answer with suitable calculations. You must show the correct values substituted into appropriate formulas.

2010 HSC
Q25

(d) Mark needs $8000 to go on a holiday in three years time. He has a 'Holiday Savings Account' with a balance of $600. **4**

He arranges to deposit $150 into this account at the end of each month for the next three years.

He earns 6% per annum interest on the money in his account, compounded monthly.

Will Mark have enough money for his trip at the end of three years? Justify your answer with suitable calculations.

17 Sally decides to put \$100 per week into her superannuation fund. The interest rate quoted is 8% per annum, compounded weekly.

Which expession will calculate the future value of her superannuation at the end of 35 years?

(A) $100\left\{\dfrac{\left(1+\dfrac{0.08}{52}\right)^{35}-1}{\dfrac{0.08}{52}}\right\}$

(B) $100\left\{\dfrac{(1+0.08)^{35}-1}{0.08}\right\}$

(C) $100\left\{\dfrac{\left(1+\dfrac{0.08}{52}\right)^{1820}-1}{\dfrac{0.08}{52}}\right\}$

(D) $100\left\{\dfrac{(1+0.08)^{1820}-1}{0.08}\right\}$

Q26

(c) Margaret borrowed \$300 000 to buy an apartment. The interest rate is 6% per annum, compounded monthly. The repayments were set by the bank at \$2200 per month for 20 years.

The loan balance sheet shows the interest charged and the balance owing for the first month.

Month	*Principal at the start of the month*	*Monthly interest*	*Monthly repayment*	*Balance at end of month*
1	\$300 000	\$1500	\$2200	\$299 300
2	\$299 300	***A***	\$2200	***B***

(i) What is the total amount that is to be paid for this loan over the 20 years? **1**

(ii) Find the values of ***A*** and ***B***. **2**

(iii) Margaret knows that she can check the bank's calculations by using the present value of an annuity formula to calculate the monthly repayment.

(1) Write down the present value of an annuity formula with the correct substitutions for this home loan. **1**

(2) Use this formula to find the calculated monthly repayment. **1**

Q27

(a) The table shows the future value of a $1 annuity at different interest rates over different numbers of time periods.

Future values of a $1 annuity

Time Period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(i) What would be the future value of a $5000 per year annuity at 3% per annum for 6 years, with interest compounding yearly? **1**

(ii) What is the value of an annuity that would provide a future value of $407 100 after 7 years at 5% per annum compound interest? **1**

(iii) An annuity of $1000 per quarter is invested at 4% per annum, compounded quarterly for 2 years. What will be the amount of interest earned? **3**